DETECTING SHIFT IN A WEIBULL PROCESS USING BOOTSTRAP METHOD

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Bachelor of Technology
In
Mechanical Engineering

By

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&
NIKESH KUMAR BEHERA

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CERTIFICATE

This is to certify that the thesis entitled, “detecting shift in a Weibull process using bootstrap method” submitted by Sri Ashwin kumar Bagh & Sri Nikesh kumar Behera in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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ACKNOWLEDGEMENT

I would like to articulate my deep gratitude to my project guide Dr. S.K. Patel who has always been my motivation for carrying out the project.

It is my pleasure to refer Microsoft word 2003 of which the compilation of this report would have been impossible.

An assemblage of this nature could never have been attempted without reference to and inspiration from the works of others whose details are mentioned in reference section. I acknowledge my indebtedness to all of them.

Last but not the least to all of my friends who were patiently extended all sorts of help for accomplishing this undertaking.

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CONTENTS

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ABSTRACT

Detecting a shift in a Weibull process is considered. The parametric bootstrap method is used to establish lower and upper control limits for monitoring percentiles.

When process measurements have a weibull distribution small percentile is of importance when observing tensile strength and it is desirable to detect their downward shift. The performance of the proposed bootstrap percentile charts is considered based on computer simulations, and some comparison is made with the existing Weibull percentile chart. The advantage of the bootstrap method is that they are not restricted by assumptions on the distribution of the process measurements. The bootstrap method uses only the sample data to estimate the sampling distribution of the parameter estimator and then determine appropriate control limit.

The new bootstrap chart indicates a shift in the process percentile substantially quicker than the previously existing chart, while maintaining comparable average run lengths when the process is in control.

Chapter 1

INTRODUCTION

Background

Objective

INTRODUCTION

The well-known Shewhart type control charts are commonly used to monitor and detect shifts in the mean or variance of a quality characteristics of interests in a process. The usual Shewhart X-bar & R control chart assume that the observed process data come from near normal distribution. However, when the distribution of the process under observation is un-known or non-normal, the sampling distribution of a parameter estimator may not be available theoretically. In this case, computational methods, such as parametric or non-parametric bootstrap method can be employed to set control limits for an appropriate control chart. An advantage of the bootstrap method is that they are not restricted by assumptions on the distribution of process measurements. The computational time, the method often requires is perhaps a perceived disadvantage, but actually is not given the current computing power available and considering the possibility of using a appropriate control chart. The bootstrap method uses only the sample data to estimate the sampling distribution of the parameter estimator and then to determine appropriate control limits. Only the usual assumptions(for phase II of the statistical process control paradigm)that the process is stable and sub-group observations are independent(identically distributed)are required.

Here, we use the parametric bootstrap method to construct control chart limits for monitoring a specified percentile of the distribution of the process characteristics of interest, e.g. the tensile strength of the material. Although the method applies more generally, we will be concerned specifically with small percentiles of the weibull distribution.

The probability density function of the weibull distribution is typically written in the form,

$F(w)=\delta/\beta*(w/\beta)^{\delta-1} * \exp [- (w/\beta)]^{\delta}$, $(w > 0, \beta, \delta> 0)$

δ= shape parameter

β= scale parameter

The Weibull density can take a variety of shapes and the exponential distribution is a special case when shape parameter is one. The 100pth percentile of the weibull distribution is,

$$Wp = \beta [-\ln (1-P)]^{1/\delta}$$

Since weibull distribution is asymmetric, when the quality characteristic of interest is a lower percentile whose estimator has a non-normal distribution, the x& R Shewhart control charts may fail to detect important shifts in the specific lower percentile in question.

BOOTSTRAP CONTROL CHART

The non-parametric bootstrap method can be applied to control charts, thereby eliminating the necessity of parametric traditional assumptions. e.g. the bootstrap X-bar chart control, but not the assumption of normally distributed process data. Bootstrap methods can be used when the distribution of statistic used to monitor the process is not available.

Bajgier developed a bootstrap control chart for the process mean, which is a competitor to Shewhart 's X-bar chart, and is appropriate when the process data may not be normal. This bootstrap control chart assumes only that the process is stable and in control when the control limits are calculated. If the assumptions is not satisfied when the control limits are computed, the limits will be too high, regardless of the distribution of the process variable. In an attempt to eliminate the assumptions required by Bajgier`s method(that the process was stable and in control when the control limits were computed), Seppala et al propped the sub-group bootstrap method, which uses observations from `k` independent sub-groups of size `n` from the process and calculates the within sub-group residuals by subtracting the mean of each group from each observation within the sub-group. Then a bootstrap sample of size `n` is taken from these residuals, a correction factor is used to adjust the variance of these resampled sub-groups, and the mean of the sub-group. Residual is added to the grand mean The bootstrap estimates are then sorted and the appropriate upper control limit(UCL)and the lower control limit (LCL)are found. Liu & Tang extended X-bar bootstrap control charts to both dependent and independent observations. Jones & Woodall evaluated the performance of these three proposed bootstrap charts and concluded that in general they did not perform substantially better than the standard methods in terms of the in control Average Run Length(ARL),noting that the work of Seppala et al was flawed. However, they indicated that for skewed distributions, bootstrap techniques seemed to yield better estimates of the true percentile values on average then Shewhart methods. Parametric bootstrap approaches produce good lower confidence bounds on weibull percentiles.

Since the weibull percentiles for quality characteristics of interest such as the breaking strength of the brittle materials, the standard X-bar & R Shewhart control charts too often may fail to detect important shifts in a specific lower percentile of interest. We use the parametric bootstrap (percentile) method to construct control charts for percentiles of the weibull distribution

and then comparing the performance of the resulting bootstrap control chart to the Padgett-Spurrier Shewhart type charts for weibull percentiles.

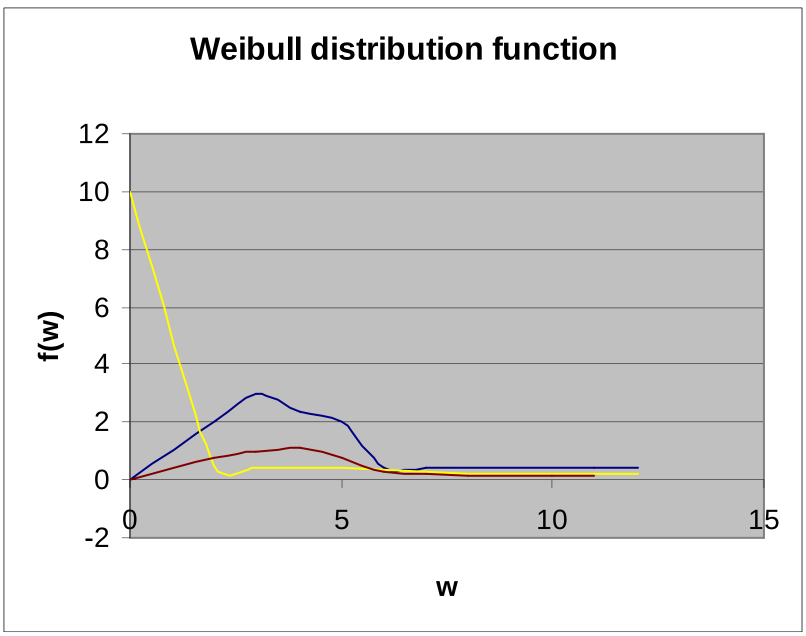


Fig. 1.1

chapter 2

WEIBULL PARAMETERS

Objective

uses

The weibull distribution has two parameters:-

a) SHAPE PARAMETER
b) SCALE PARAMETER

After we give the parameters the weibull distribution program produces the mean & variance for the distribution we have given. If an x-parameter is additionally given the program returns cumulative probability and the density values, if we give cumulative probability value the program returns appropriate \`x\` value. The exponential distribution(used to study waiting time)is a special case of the weibull distribution with alpha =1, mean = beta, and lambda(the hazard rate) =1/beta.Another special case of the weibull distribution is the Rayleigh distribution(used to study wind speed or to make certain transformation).For Rayleigh\`s distribution alpha is fixed at 2.

In practice weibull distribution is used to describe two groups of phenomena. The lifetime of objects method is used in quality control. A manufacturer provides the weibull parameter for a product and the user can calculate the probability that a part fails after one, two, three or many years. The weibull distribution program allows us to do these calculation on the basis of already known parameters e.g. if we want to know the proportion that fails after one, two or many years, enter the value on the \`x\` box & read out the cumulative probability value. If we want to know the moment in time at which half the parts will have failed, enter the value 0.5 in % box and read out the \`x\` value. The description of wind speed is an example of the use of weibull distribution to describe natural phenomena. Each part of planet has its own parameter for a weibull distribution to describe the wind speed pattern in that place. On the basis of that we can calculate the no. of days of a year, with wind speed over a certain force, or the mean wind speed or the median wind speed. Half the days in a year have wind speed below the median force & halve the days above. The distribution does not allow negative values and it is easy to appropriately
consider the fact that on most days there will be a bit of wind, some days more some days way too much.

USES

The weibull distribution is most commonly used in life data analysis, though it has found applications as well. The Weibull distribution is often used in place of the normal distribution due to a fact that weibull variation can be generated using the more complicated `box-muller` method which requires two uniform random variates. This distribution may also be used to represent manufacturing and delivery time in industrial engineering problems, while it is very important in extreme value theory and weather forecasting. It is also a very popular statistical model in reliability engineering and failure analysis while it is widely applied in radar systems to model the dispersion of the received signal levels produced by some type of clutters. Furthermore, concerning wireless communications, the weibull distribution may be used for fading channel modeling. Since the weibull fading model seems to exhibit good but to experimental fading channel measurements. The weibull distribution is also commonly used to describe wind speed distribution as the natural distribution often matches the weibull shape.

Chapter 3

CHARACTERISTICS OF WEIBULL PARAMETER

Effects

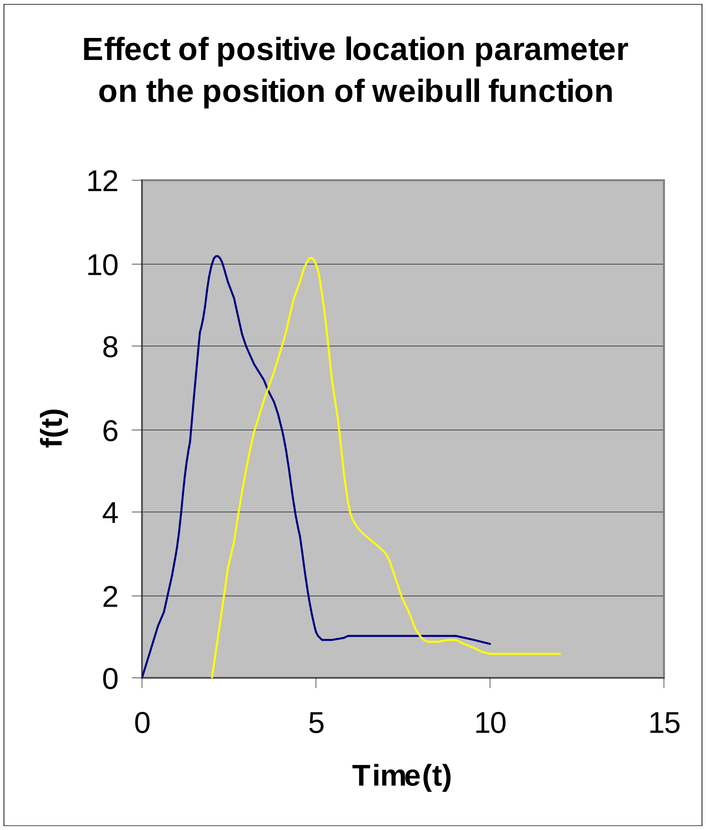


Fig.3.1

a) when g =0, the distribution starts at t =0 or at the origin.

b) If g >0, the distribution starts at the location `g` to the right of the margin.

c) If g <0, the distribution starts at the location `g` to the left of the margin.

d) `g` provides an estimate of the earliest time to failure of such units.

e) The life period 0 to +g is a failure free operating period of such units.

f) The parameter `g` may assume all values and provide an estimate of the earliest time, a failure may be observed. A negative `g` may indicate that failures have occurred prior to the beginning of the test, namely during production, storage, in transit, during checkout prior to the start of a mission or prior to actual use.

g) `g` has the same units as `T`, such as hours, miles, cycles fluctuations.

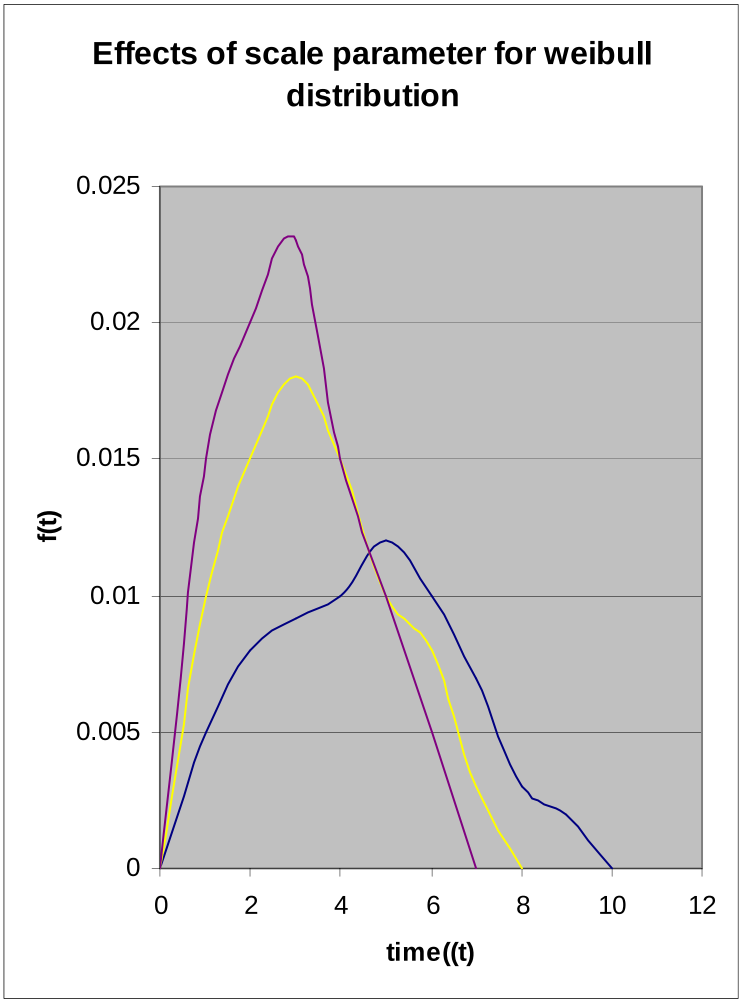


Fig 3.2

The change in the scale parameter `β ` has the same effect as a change of abscissa scale.

a) if `β ` is increased while ` δ` & `g ` are kept the same, the distribution gets stretched out to the right and its height decreases, while maintaining its shape &location.
b) If `β ` is decreased while `δ ` & `g` are kept the same, the distribution gets pushed in towards left(towards o or g) and its height increases.
c) ` β` has the same unit as T such as hours, miles, cycles, actuations etc.

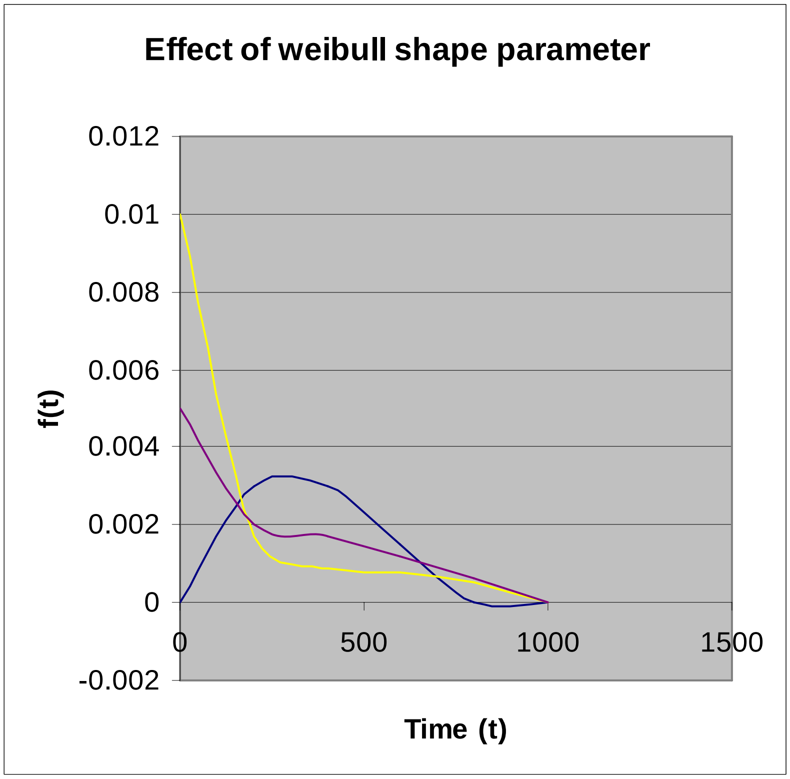


Fig 3.3

For $0<\beta\leq 1$

- As $T \to 0$ (or g), $f(t) \to \infty$
- As $T \to \infty$, $f(t) \to 0$
- F(t) decreases monotonically and is convex as `T` increases beyond the value of `g`
- The mode is non-existent

For $\delta > 1$

- $f(T) = 0$ at $T = 0$ (or g)
- f(T) increases as $T \to \tau$(mode) and decreases thereafter.
- For $\delta < 2.6$ the weibull pdf is positively skewed (has a right tail)
- For $2.6 < \delta < 3.7$, its co-efficient of skewness approaches zero (no tail). Consequently, it may approximate the normal pdf .
- For $\delta > 3.7$, it is negatively skewed (left tail)

Chapter 4

Control charts

Introduction

Objectives

Control chart

A control chart is a graphical representation of the collected information. The information may pertain to measured quality characteristics or judged quality characteristics of samples. It detects the variation in processing and warns if there is any departure from the specified tolerance limits.

In other words control chart is a device which specifies the state of statistical control, a device for attaining statistical control, a device to judge whether statistical control has been attained. The control limits on the chart are so placed as to disclose the presence or absence of assignable causes quality variation. This makes possible the diagnosis and correction of many production troubles and often brings substantial improvements in product identifying certain of the quality variations as in-evitable chance variations, the control chart tells when to leave the chart alone and thus prevents unnecessary frequent adjustments that tend to increase the variability of the process rather than to decrease it. With the help of control chart it is possible to find out the natural capability of a production process which permits better decisions on engineering tolerances and better comparison between alterative design and alternative production methods. Through improvement of conventional acceptance procedure it often provides better quality assurance at low inspection cost. There are many types of control chart designed for different control situation, each with its own advantages and disadvantages and with its own field of applications. Control charts which are most commonly used are:-

- Control charts for variables (X-bar & R chart)
- Control chart for fraction defective (p- chart)
- Control chart for no. of defects per unit(c- chart)

Control charts for variables are useful for controlling fully automatic process where the operator is probably responsible for three or more machines.

Control charts for fraction defective and defects per unit are attribute control charts. A fraction defective control chart discloses erratic fluctuation in the quality of inspection which may result in improvement in inspection practice and inspection standards.

OBJECTIVES

X-bar &R or X-bar & σ charts are used in combination for the control process. X-bar chart shows the centering of the process i.e. it shows the variation in the average of the samples. R – chart shows the uniformity or consistency of the process i.e. it shows the variation in the ranges of the samples. It is a chart for measure of spread. σ chart shows the variation of the process.

- The control charts are used to determine whether the given process can meet existing specifications without a fundamental change in the production process. In other words they tell the process is in control and if so at what dispersion.

- To secure the information used in establishing or changing production procedures. Such change may be either elimination of assignable causes of variation that may be called for whenever the control chart makes it clear that specifications cannot be met with present methods e.g. where both upper and lower values are specified for a quality characteristics, as in the case of dimensional tolerances. If the basic variability of the process is so great that it is impossible to make all the product within the specification limit and the specification cannot be changed.

1) To make a fundamental change in production process that will reduce the basic variability.

2) To suffer and sort out the good(non-defective) for the bad (defective products).

- To secure information when it is needed to widen the tolerances. Sometimes the control chart shows so much basic variability that some products are sure to be made outside this tolerance, a review of the situation will show that the tolerances are higher than necessary for the functioning of the product. Therefore the appropriate action will be to change the specifications to widen the tolerances for the sake of economy.
- To secure information to be used in establishing or changing inspection procedure or acceptance procedure or both.
- To provide a basis for current decisions on acceptance or rejection of manufactured or purchased product. It is possible to reduce inspection cost by using the control chart for variable for acceptance.
- To provide a basis for current decisions during production as to when to hunt for causes of variation and take action so as to correct them and when to leave a process alone.

WEIBULL PERCENTILE CONTROL CHART

It is unclear if the standard X-bar & R charts which have the correct error probabilities or if they will correctly signal a shift in the 100pth percentile, Wp of the weibull distribution. In both the in-control & out of control cases ,the X & R chart can signal out of control much too soon or not soon enough. This was illustrated by Padgett & Spurrier for weibull distribution.

The parametric bootstrap method is used to develop control chart for small percentile of the weibull distribution. The method works for any percentile, but as mentioned earlier lower percentile are of particular interest for tensile strength consideration. An advantage of the bootstrap control chart of the weibull percentile is that tabled co-efficient required for the BLIEs used by Padgett & Spurrier are not needed. The behaviour of the bootstrap weibull percentile control limits the performance of the proposed bootstrap weibull percentile chart and the comparison with the existing control chart will be investigated briefly by computer simulations.

Chapter 5

COMPUTER PROGRAM

Generate data randomly for weibull distribution randomly

Show the graph of weibull distribution function

GENERATE DATA RANDOMLY FOR THE WEIBULL DISTRIBUTION FUNCTION

```
# include ‹ stdio.h ›
# include ‹ conio.h ›
# include ‹ math.h ›
# include ‹ stdlib.h ›

void main ( )

  {
          float del, beta, w,  f, temp;
          int  more = 1;
          clrscr (  );
          while ( more = = 1 )
            {
              printf  (" enter values of  delta and beta \n" );
              w = random ( 10000 );
              w = (w/10000);
              printf ( "values of w generated = % f \n" ,w );
              temp = exp ( pow ( w/beta), del );
              f = (( del/beta)* pow (( w/beta),(del – 1)))/temp;
              printf (" %f \n" , f );
              printf (" more values of the function ( 1/ 0 )" );
              scanf (" %d ", & more );
            }
getch ( );
}
```

SHOW THE GRAPH OF THE WEIBULL DISTRIBUTION FUNCTION

```
# include ‹ stdio.h ›
# include ‹ conio.h ›
# include ‹ stdlib.h ›
# include ‹ graphics.h ›

void main ( )
{

                    double del = 4, beta = 1, w[20], f[20],temp;
                    int = 0;
                    int gd = DETECT ,gm, x, y;
                    clrscr ( );
                    while ( i < 20 )
                      {
                          w[i] = random ( 6 );
                          printf (" value of w generated, %f \n", w[i] );
                          temp = exp ( pow (( w[i]/beta), del);
                          f[i] = (( del/beta )* pow (( w[i]/beta), ( del-1 )/temp;
                          printf (" %f\n ",  f[i] );
                          i++;
                       }
                        getch ( );
                          intigraph ( &gd, &gm, "c:\\tc\\bgi" );
                          line (10, 410, 510, 410);
                          line ( 10, 40, 10, 10);
                          for ( i = 0; i < 20; i++ )
```

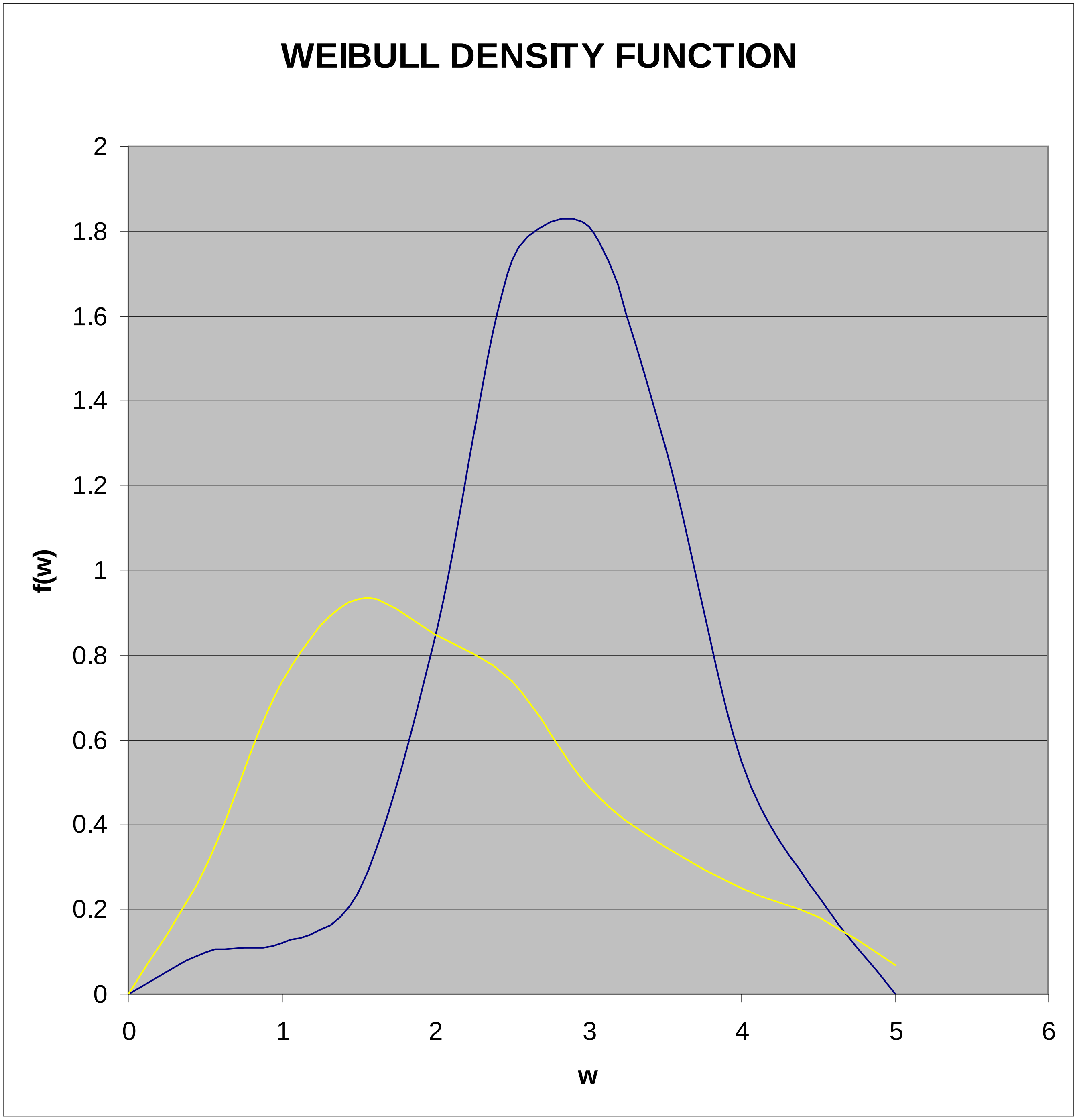
```
{
    x = 100 * w[i];
    y = 50 * f[i]* 4;
    circle (x + 10, 410 – y, 2);
 }
getch ( );
}
```

GENERATED VALUE FOR SHAPE PARAMETER 4.8 & SCALE PARAMETER 3.2

Sub-groups	1	2	3	4	5
1	3.70	2.74	2.73	2.50	3.60
2	3.11	3.27	2.87	1.47	3.11
3	4.42	2.41	3.19	3.22	1.69
4	3.28	3.09	1.87	3.15	4.90
5	3.75	2.43	2.95	2.97	3.39
6	2.96	2.53	2.69	2.93	3.22
7	3.39	2.81	4.20	3.33	2.55
8	3.31	3.31	2.85	2.56	3.56
9	3.15	2.35	2.55	2.59	2.38
10	2.81	2.77	2.17	2.83	1.92

GENERATED VALUE FOR SHAPE PARAMETER 2.0 & SCALE PARAMETER 2.6

SUBGROUP	1	2	3	4	5
11	1.41	3.68	2.97	1.36	0.98
12	2.76	4.91	3.68	1.84	1.59
13	3.19	1.57	0.81	5.56	1.73
14	1.59	2.00	1.22	1.12	1.71
15	2.17	1.17	5.08	3.48	1.18
16	3.51	2.17	1.69	1.25	4.38
17	1.84	0.39	3.68	2.48	0.85
18	1.61	2.79	4.74	2.03	1.80
19	1.57	1.08	2.03	1.61	3.12
20	1.89	2.88	2.82	2.05	3.65



WEIBULL DENSITY FUNCTION
f(w)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
w

Fig 5.1

$\delta=[(\sum^{k}_{j=1}\sum^{n}_{i=1}x_{ij}^{\delta}\log x_i / \sum^{k}_{j=1}\sum^{n}_{k=1}x_{ij}^{\delta})-(\sum^{k}_{j=1}\sum^{n}_{i=1}x_{ij}^{\delta}\log x_{ij}/nk)]$

$\beta=[(\sum^{k}_{j=1}\sum^{n}_{i=1}x_{ij}^{\delta}/nk)]^{1/\delta}$

$w_p=\beta(-\ln(1-p))^{1/\delta}$

$\delta=[3.7^{4.8}\log 3.7 + 2.74^{4.8}\log 2.74 +\ldots\ldots+1.92^{4.8}\log 1.92/3.7^{4.8}+2.74^{4.8+}\ldots\ldots+ 1.92^{4.8}]$
$- [\log 3.7+\log 2.74+\ldots\ldots+ \log 1.92/5*10]$

$=4.78$

$\beta= 3.7^{4.8}+2.74^{4.8}+\ldots\ldots+1.92^{4.8}/5*10]^{1/4.8}$

$=3.2$

$W_p = 3.2 * (-\ln (1- .01))^{1/4.78}$

$=1.227$ (for $p = 1\%$)

Similarly for all the sub-groups b, δ & w_p values were determined for both the observations. By determining the upper control limit (UCL) and the lower control limit (LCL) the following graph was plotted which shows the shift in the process. The LCL & UCL values were found to be 0.40 & 2.39 respectively.

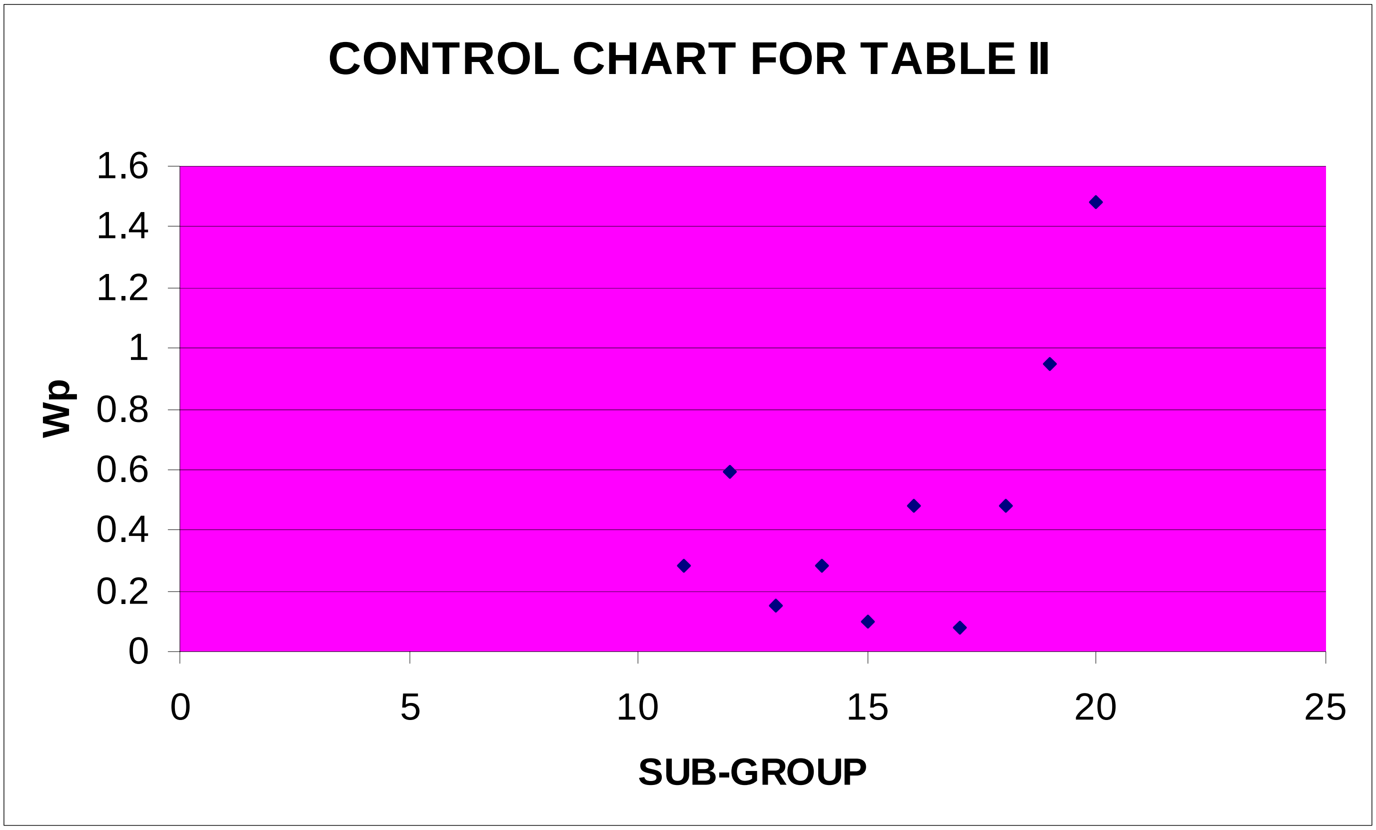

CONTROL CHART FOR TABLE II
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Wp
0
5
10
15
20
25
SUB-GROUP

Fig 5.2

Chapter 6

CONCLUSION

CONCLUSION

The use of parametric bootstrap control charts clearly has advantages when process data come from skewed distributions and/or atypical parameters such as percentiles are to be monitored. Construction of such charts should not be problematic with current computing power available, and should be used especially when standard charts, such as X-bar or R charts are known to be applicable in these situations.

An advantage of the bootstrap chart for Weibull percentiles is that the tabled coefficients required for the BLIES used by Padgett and Spurrier are needed. The concern is with process monitoring after process has been stabilized and it has been justified that the observed values from the process are from a Weibull population, at least approximately. Weibull probability plots of observed sample data from the process can be used from the latter.

Considering a process in detecting the shift in the tensile strength the parameter of interest is the first percentile of the tensile strength distribution. A decrease in the first percentile would indicate a decrease in tensile strength, and a control chart can be used to detect such changes. The Weibull distribution is a reasonable model for the tensile strength of such materials.

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